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NEL REPORT 1188

ESTIMATING THE ACCURACY OF NAVIGATION SYSTEMS

A statistical method of determining circular error probable

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Research Report 1188

24 October 1963

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THE PROBLEM

Devise a statistical method of estimating the fix accuracy of a navigation system.

RESULTS

1. A statistical method involving system geometry and a simple estimate of the statistical repeatability of measurement was developed to determine the circular error probable in a navigational fix (not, however, accounting for fixed errors such as map errors).

2. The method which was developed is directly applicable to the design or evaluation of hyperbolic systems and has been successfully used in evaluation studies of the present Omega network. An example of this application is presented.

ADMINISTRATIVE INFORMATION

Work was performed in March 1962 under SS 161 001, Task 6101 (NEL A1-4). The report was approved for publication 24 October 1963.

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INTRODUCTION

This report describes a statistical method of estimating the fix accuracy of a navigation system. The method is directly applicable to the design or evaluation of hyperbolic systems such as Omega and Loran. It consists of applying a knowledge of system geometry and a simple estimate of the statistical repeatability of measurement to find the circular error probable in a navigational fix. The circular error so determined should be used with caution because it does not account for fixed errors such as map errors.

The method described herein has been successfully used in evaluation studies of the present Omega network. This report contains an illustrative example of this application.

FIX ACCURACY OF HYPERBOLIC NAVIGATION SYSTEMS

Theoretical Approach

In developing a mathematical model for estimating the fix accuracy of a navigation system, it was assumed that the basic system geometry would be either known or easily computed from known transmitter coordinates -- that is, the crossing angle and divergence of lines of position. Further necessary assumptions, which may be made from a study of propagation and the equipment specifications, concern propagation effects and random variations due to noise in the receiving or transmitting equipment. Exclusive of propagation effects the standard deviations of measurements in time units on the applicable lines of position and their correlation would be constant throughout the

field. In this case,

$$\sigma_A = \Omega_A \sigma'_A = \text{Standard deviation of line of position } A \text{ (distance units)}$$

$$\sigma_B = \Omega_B \sigma'_B = \text{Standard deviation of line of position } B \text{ (distance units)}$$

$$\theta = \text{Crossing angle of lines of position (measured from } A \text{ to } B)$$

$$\rho_{AB} = \rho'_{AB} = \text{Correlation coefficient between line-of-position measurements}$$

where

$$\Omega_j = \text{Divergence of line-of-position } j$$

$$\sigma_j = \text{Standard deviation of line-of-position } j \text{ (time units)}$$

would be known. In most practical cases, propagation effects are significant to a degree where σ'_A , σ'_B , and ρ'_{AB} are not constant but may also vary with position, in some cases diurnally. Two separate problems may therefore exist: (1) determining σ'_A , σ'_B , and ρ'_{AB} as functions of position, time, etc., and (2) computing the median fix error $R(\sigma_A, \sigma_B, \rho, \theta)$. An additional complication is caused by fixed or bias errors giving rise to "offset" distributions.

Distortions, Misadjustments, and Fixed Errors

Although fixed-equipment errors and propagation-associated field distortions unremoved by appropriate theory are not considered, it is important to note how such errors affect the applicability of the accuracy estimate. While fixed errors should be considered an evaluation of any system, their effect at any specified point cannot be determined from general considerations. The restriction of solely considering random variables will then yield information only on the median of the distribution function

of latitude-longitude points which may or may not be the median fix error.

Field distortions and equipment errors are likely to be present to a significant degree in any practical system, and will result in a mean fix being obtained that is not at the true position. Statistics for "offset" distributions of this type have been considered, but it is uncertain as to what meaning can be attached to the results. To provide a definite answer, the magnitude and direction of the offset must be known. If the offset were known, it would be removed. The following example is typical of the effects of fixed errors:

A system exists for which the mean field distortion over the total coverage area is 0.4 mile. At a given point the median repeatability is 0.4 mile. What is the probability of obtaining a fix at the given point within 0.6 mile?

This question cannot be answered with the information given since the field distortion at the point was never specified. It might be zero, in which case the probability would be about 0.87. Alternatively, the field distortion might be 3 miles, in which case the probability would be small. Thus, if fixed errors are present at any given point in a system, no answer is necessarily correct. Hence, while the presence of offsets as, say, estimated by the RMS fix error, is of importance to any system, it cannot be used to estimate accuracy at a prescribed point such as one may obtain by the following analysis.

Circular Error Probable

The general approach will be to consider a normal bivariate distribution specified by the parameters σ'_A , σ'_B , and ρ'_{AB} , which apply in a skewed, linear coordinate system with crossing angle θ and scale factors Ω_A and Ω_B , as diagrammed in figure 1.

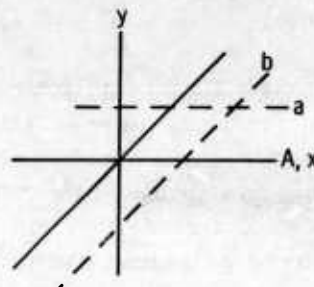


Figure 1. The coordinate system.

With proper choice of parameters this is the linear approximation to any point in a hyperbolic grid. A rotation of the B axis of $(90^\circ - \theta)$ will be made first to obtain a correlated, normal bivariate distribution in rectangular coordinates (x, y) . The (x, y) coordinate system will then be rotated through an angle α to obtain stochastically independent variables. The circular error probable may then be computed from the parameters of this distribution. The axis variations are shown in figure 2.

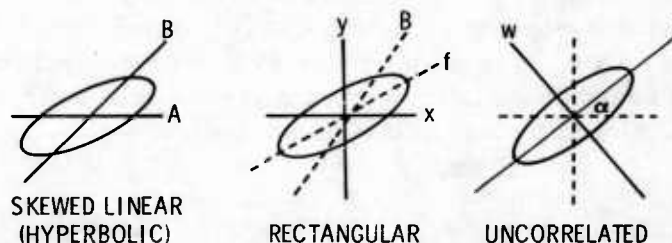


Figure 2. Coordinate transforms.

The principal task will be to determine the transforms for the standard deviations and correlation coefficient throughout the various coordinate transforms.

The transformation from the skewed system to the linear system is readily accomplished if the scales are first normalized by

$$a = \Omega_A a' \text{ and } b = \Omega_B b' \quad (1)$$

By definition

$$\text{Coordinate } y = \text{value of line-of-position } A = a \quad (2)$$

Note that a is the line-of-position value and not the displacement along the coordinate A . Geometry yields

$$x = a \operatorname{ctn} \theta + b / \sin \theta \quad (3)$$

Transforms for the means, standard deviations, and correlation coefficient are now required. Obviously

$$\bar{y} = \bar{A} \quad (4)$$

$$\sigma_y = \sigma_A \quad (5)$$

By definition*

$$\begin{aligned} \bar{x} &= \frac{\sum x_i}{n} \\ \bar{x} &= \frac{\sum \left(\frac{b_i}{\sin \theta} + a_i \operatorname{ctn} \theta \right)}{n} \\ \bar{x} &= \frac{1}{\sin \theta} \frac{\sum b_i}{n} + \operatorname{ctn} \theta \frac{\sum a_i}{n} \\ \bar{x} &= \frac{\bar{B}}{\sin \theta} + \bar{A} \operatorname{ctn} \theta \end{aligned} \quad (6)$$

* Simple nomenclature has been used without specific reference to the algebra of expectations.

By definition

$$\sigma_x^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

and

$$\sigma_A^2 = \frac{\sum a_i^2}{n} - \bar{A}^2; \quad \sigma_B^2 = \frac{\sum b_i^2}{n} - \bar{B}^2$$

and

$$\rho_{AB} = \frac{\frac{\sum a_i b_i}{n} - \bar{A} \bar{B}}{\sigma_A \sigma_B}$$

which may be combined and simplified by using (5) and (4) and (6)

$$\begin{aligned} \sigma_x^2 &= \frac{\sum \left(\frac{b_i}{\sin \theta} + a_i \operatorname{ctn} \theta \right)^2}{n} - \left(\frac{\bar{B}^2}{\sin^2 \theta} + 2\bar{A}\bar{B} \frac{\cos \theta}{\sin^2 \theta} + \bar{A}^2 \operatorname{ctn}^2 \theta \right) \\ \sigma_x^2 &= \frac{1}{\sin^2 \theta} \left(\frac{\sum b_i^2}{n} - \bar{B}^2 \right) + \frac{2 \cos \theta}{\sin^2 \theta} \left(\frac{\sum a_i b_i}{n} - \bar{A} \bar{B} \right) + \operatorname{ctn}^2 \theta \left(\frac{\sum a_i^2}{n} - \bar{A}^2 \right) \\ \sigma_x^2 &= \frac{\sigma_B^2}{\sin^2 \theta} + 2 \sigma_A \sigma_B \rho_{AB} \frac{\cos \theta}{\sin^2 \theta} + \sigma_A^2 \operatorname{ctn}^2 \theta \end{aligned} \quad (7)$$

In a similar manner

$$\rho_{xy} = \frac{\left(\frac{\sum x_i y_i}{n} - \bar{x} \bar{y} \right)}{\sigma_x \sigma_y}$$

$$\rho_{xy} = \left(\frac{\sigma_A}{\sigma_B} \cos \theta + \rho_{AB} \right) \left[1 + 2 \rho_{AB} \left(\frac{\sigma_A}{\sigma_B} \right) \cos \theta + \left(\frac{\sigma_A}{\sigma_B} \right)^2 \cos^2 \theta \right]^{-\frac{1}{2}} \quad (8)$$

which completes the first phase of transformation.

Transformations to the uncorrelated form are given in Hald.¹ In particular

$$\sigma_f \sigma_w = \sigma_x \sigma_y \sqrt{1 - \rho_{xy}^2} \quad (9)$$

$$\sigma_f^2 + \sigma_w^2 = \sigma_x^2 + \sigma_y^2 \quad (10)$$

which we wish to solve for σ_f , the standard deviation on the major axis, and $C = \sigma_w / \sigma_f$. Note that except in the degenerate case σ_w is a unique function of σ_f and conversely. In particular

$$C = \frac{\sigma_x \sigma_y}{\sigma_f^2} \sqrt{1 - \rho_{xy}^2} \quad (11)$$

σ_f is given by the solution of the degenerate fourth degree equation obtained from (9) and (10)

$$\sigma_f = \frac{\sigma_x + \sigma_y^2}{2} \sqrt{1 + \sqrt{1 - \frac{4\sigma_x^2 \sigma_y^2 (1 - \rho_{xy}^2)}{(\sigma_x^2 + \sigma_y^2)^2}}} \quad (12)$$

where two choices of sign have been made by demanding that σ_f be positive. The remaining sign is chosen so that $C \leq 1$.

No analytic solution has been found for obtaining the radius of a circle containing a given probability for a general bivariate distribution. Numerical methods have been applied² such that the function $R(C, \sigma_f, P)$ is known for

¹Hald, A., Statistical Theory, With Engineering Applications, p. 596-599, Wiley, 1952

²Harter, H. L., "Circular Error Probabilities," American Statistical Association. Journal, v. 55, p. 723-731, December 1960

particular values of the probability P . Hence, for the circular error probable, median, $P = 0.5$, we may determine R^*

$$R = R(C, \sigma_f, 0.5) = K(C, 0.5) \sigma_f \quad (13)$$

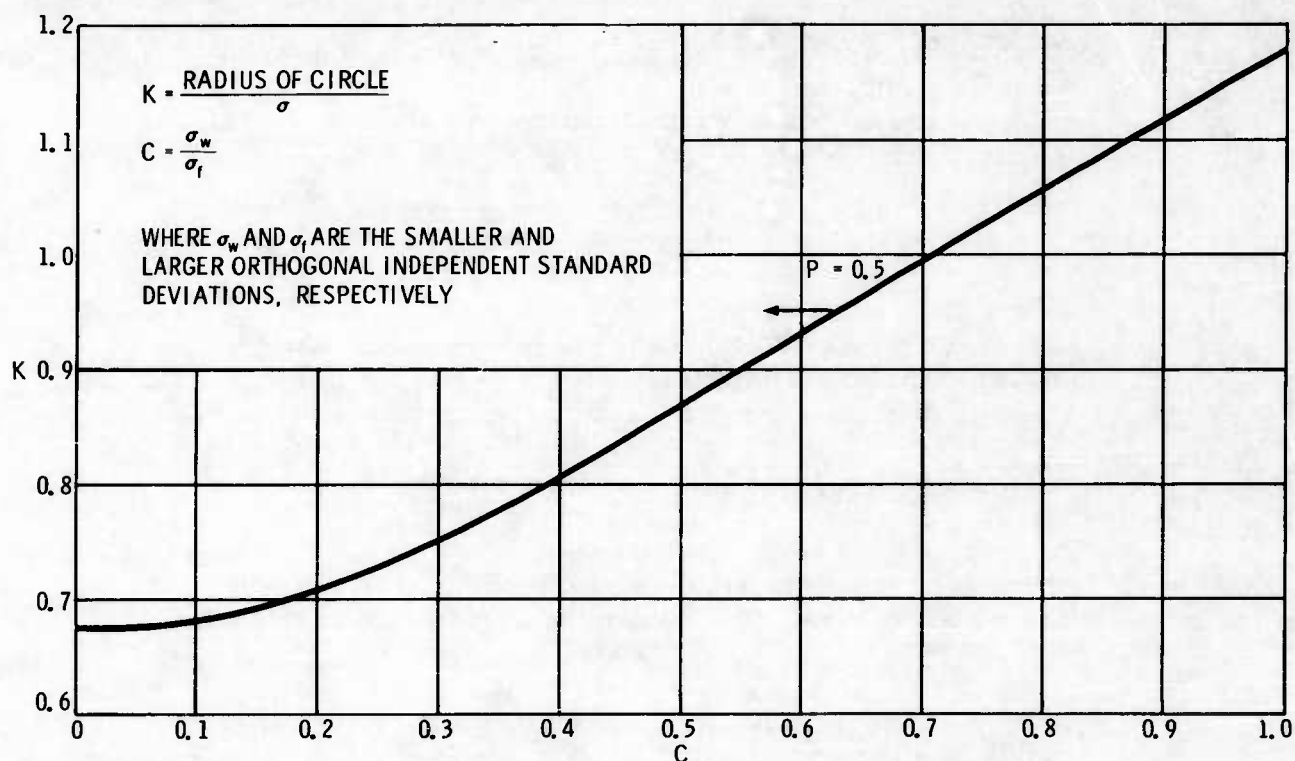


Figure 3. Radius of a circle containing probabilities of 0.5 for a bivariate system (plotted from ref. 2).

*Ref. 2, p. 725-26, provides an extensive table of $P(K, C)$ from which $K(P, C)$ may be obtained (p. 728). The function $K(C, 0.5)$ is plotted in figure 3 from this reference.

Combining equations 1, 5, 7, 8, 11, 12, and 13 then yields the desired function

$$R(\sigma'_A, \sigma'_B, \Omega_A, \Omega_B, \rho_{AB}, \theta_{AB})$$

Values of R may then be calculated for representative points in the coverage area of a given system. From a spatial matrix of R values, contours of constant system accuracy can be drawn by interpolation.

This approach was used to evaluate the present experimental Omega system with its triad of stations in Hawaii, Panama, and New York State. Daytime system operation was approximated by assuming that the standard deviations on the lines of position were everywhere constant and everywhere correlated by the same amount; that is, $\sigma_A = \sigma_B = 4$ per cent and $\rho_{AB} = 0.4$ (the physical justification for these assumptions is discussed in Appendix A). The lane divergences, Ω_A and Ω_B , and the crossing angle were then obtained for various points in the coverage area either by direct computation or, more simply, from general system coverage charts.³ A hand calculator (Appendix B) provided a matrix of R values. Graphical interpolation permitted construction of an accuracy map (fig. 4).

³Navy Hydrographic Office Chart 17150, Omega System Area Coverage, November 1961

ASSUMPTIONS: STANDARD DEVIATION ON BOTH
PAIRS = 4% EVERYWHERE
CORRELATION BETWEEN LINES OF
POSITION = +0.4 EVERYWHERE

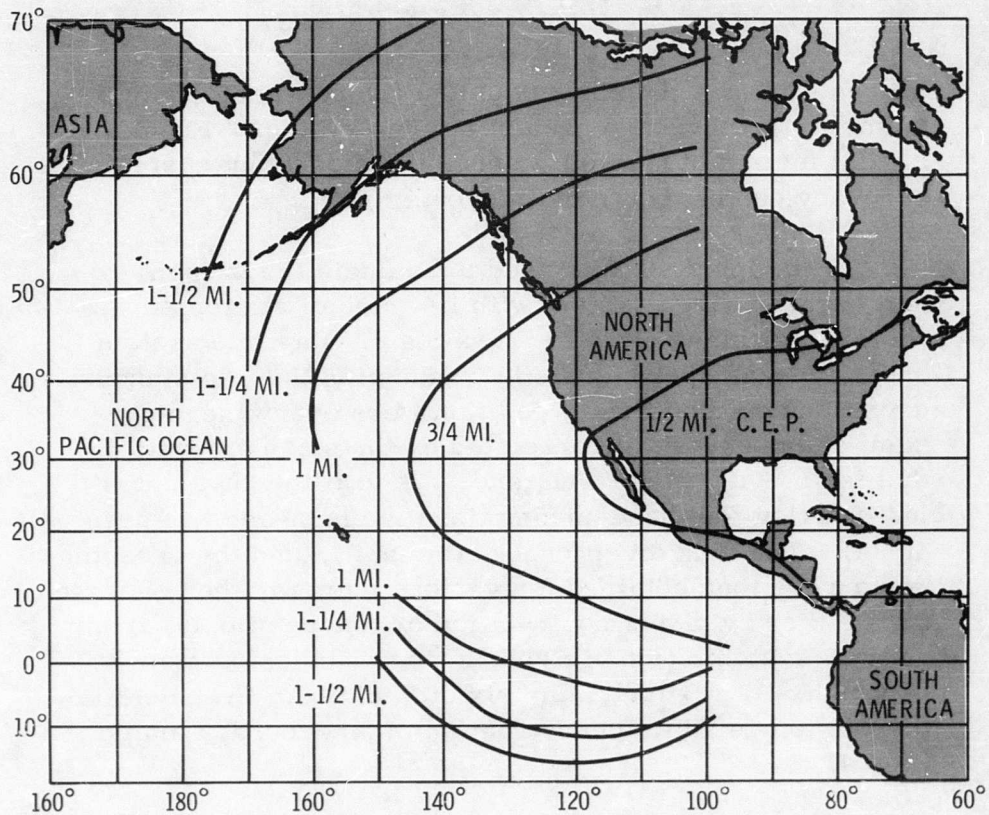


Figure 4. Omega system accuracy, daytime conditions.

SUMMARY

A rigorous means for estimating the accuracy of navigation systems has been devised, subject to reasonable estimates of parameters. The method is applicable to hyperbolic systems such as Loran and Omega. It provides a straightforward way of computing accuracy contours based on circular error probable.

APPENDIX A: PHYSICAL JUSTIFICATION OF CONSTANT σ AND ρ

It is obvious that the standard deviation to be expected in any practical system will not be constant everywhere. Locations will exist outside the coverage area where the received signal strength is inadequate for measurement. If slave stations are functioning as perfect reflectors, measurements near the transmitting antenna should yield data of low variance. However, if the field strength is adequate for reasonable measurements, greater field strength will probably not reduce significantly the variance to be observed.

The complex interaction of components of vlf propagation yields phase irregularities in the medium field, so that minor ionospheric fluctuations may produce a higher variance in measurements than would be obtained under similar conditions in the far field. The higher phase gradient in the medium field might be expected to offset to some extent conditions in the far field due to the greater propagation distance, to produce a quasi-constant variance on phase measurements.

In any event, only small spatial deviations in daytime variances have been observed. Experimental data at night are as yet unresolved on possible spatial dependence of variance. It may be noted that for an equipment-limited system the variance would not be nominally a function of position. Insufficient data have been analyzed to determine accurate values of ρ . However, the approximate value is indicated and is comparable to Loran A experience. Logically, the correlation coefficient should vary, depending in an elaborate fashion on a general ionospheric spatial correlation function (point-to-point) and the length of the common path from an observation point to the master station. In an equipment-limited system the correlation would tend to be spatially independent and assume a constant value because of the common receiver channel.

APPENDIX B: OUTLINE FOR CONVENTIONAL HAND CALCULATION

Column:	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫
Function:	Lat	Long.	σ_A'	σ_B'	ρ_{AB}	Ω_A	Ω_B	θ	$\sigma_A' \sigma_B'$	σ_B	$\sin \theta$	$\cos \theta$
Operation:	Given	Given	Given	Given	Given	Chart	Chart	Chart	③ ⑥	④ ⑦	$\sin \theta$	$\cos \theta$
Example:	30° N	100° W	4.0	4.0	0.4	0.115	0.085	112°	0.46	0.34	0.92	-0.39

Column:	⑬	⑭	⑮	⑯	⑰	⑱	⑲	⑳	㉑	㉒	㉓	㉔
Function:	$\cot \theta$	$\cot^2 \theta$	$\sin^2 \theta$	$2\sigma_A \sigma_B \rho_{AB} \sin^2 \theta$	$\sigma_A^2 \cot^2 \theta$	$\sigma_B^2 / \sin^2 \theta$	σ_x^2	σ_A / σ_B	$\frac{\sigma_A}{\sigma_B} \cos \theta$	$\frac{\sigma_A}{\sigma_B} \cos \theta + \rho_{AB}$		
Operation:	⑫ / ⑪	⑬ ⑬	⑪ ⑪	2⑤ ⑨ ⑩ ⑮	⑨ ⑨ ⑭ ⑮	⑩ ⑩ / ⑮	⑬ + ⑰ + ⑱	⑨ / ⑩	⑳ ⑫	㉑ + ⑤	㉒ ㉑	㉓ ㉑
Example:	-0.42	0.176	0.85	-0.057	0.057	0.14	0.12	1.35	-0.53	-0.13	0.28	0.8

Column:	㉕	㉖	㉗	㉘	㉙	㉚	㉛	㉜	㉝	㉞	㉟	㊱
Function:			ρ_{xy}	σ_y^2	$\sigma_x^2 + \sigma_y^2$	$(\sigma_x^2 + \sigma_y^2)^{3/2}$	$\sqrt{\sigma_x^2 + \sigma_y^2}$	ρ_{xy}^2	$1 - \rho_{xy}^2$	$4\sigma_x \sigma_y (1 - \rho_{xy}^2)$		
Operation:	㉒ ㉑ + ㉓ + 1	$\sqrt{㉒}$	㉒ / ㉒	⑨ ⑨	㉒ + ⑰	㉒ ㉒	$\sqrt{㉒}$	㉒ ㉒	1 - ㉒	4 ① ㉒ ㉒	㉒ ㉒	1 - ㉒
Example:	0.86	0.93	-0.14	0.21	0.33	0.11	0.57	0.020	0.98	0.099	0.90	0.10

Column:	㊲	㊳	㊴	㊵	㊶	㊷	㊸	㊹	㊺	㊻	㊼	㊽
Function:					σ_t			σ_t^2	C	KIC, 0.5)	R	
Operation:	$\sqrt{㉒}$	1 + ㉒	$\sqrt{㉒}$	㉒ / 1.414	㉒ ㉒	㉒ / 4	$\sqrt{㉒}$	㉒ ㉒	㉒ ㉒	Special Graph	㉒ ㉒	
Example:	0.32	1.32	1.15	0.81	0.46	0.025	0.15	0.21	0.71	1.00	0.46	

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